Introduction to Computational Mechanics

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Course description

The goal of this course is to introduce computational methods based on energy minimization to solve boundary value problems (BVPs) encountered in many engineering applications. In particular, the focus will be on the Finite Element Method and its application to linear elasticity problems. Stochastic methods such as Monte Carlo simulation commonly employed in material science and physics will also be discussed.

First, we will discuss how to discretize a continuum problem and construct a model that can be solved numerically. Subsequently, we will discuss the principle of minimum potential energy and its application to solve a mechanical system with discrete degrees of freedom.

We will then introduce the more general weak form formulation and the solution of BVPs based on the Euler equations.

Within this energetic framework, we will emphasize the single concepts essential to translate the continuum problem into a set of element equations with discrete degrees of freedom: shape functions, numerical integration, assembly, isoparametric elements.

We conclude the course with an introduction to Monte Carlo methods to solve problems characterized by multiple energy minima. As demonstration, we will apply the Monte Carlo method to a set of discrete particles interacting with pair potentials.

Prerequisites

This course is intended for upper division students. Familiarity with multi-variable calculus, matrix algebra, and basic concepts in mechanics is expected. Essential concepts will nevertheless be reviewed as necessary during lectures and lab sessions.

Syllabus

List of the topics discussed in this course include:

- Modeling and discretization of a continuum problem.
- Direct formulation of the finite element method.
- Strain energy U.
- Potential energy V due to loads.
- Solution of 1D problems using the principle of minimum potential energy.
- The one-dimensional boundary value problem.
- The weak form.
- Euler equations.
- Method of weighted residuals.

- Rayleigh-Ritz method.
- Galerkin method.
- 1D shape functions: linear and quadratic interpolation.
- Rigid body and deformational modes of displacement.
- Quadrature rules.
- Completeness, compatibility, and convergence.
- Finite element analysis using one-dimensional elements.
- 2D shape functions.
- 2D isoparametric finite elements.
- System of particles and Monte Carlo Method.

Calendar

Each class on Saturday is composed of two parts: a two-hour lecture from 9 to 11am and a two-hour laboratory from 12 to 2pm with a break between 11am and 12pm. We will meet on the following days:

1. Saturday 02/04: 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
2. Saturday 02/11 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
3. Saturday 02/18 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
4. Saturday 02/25 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
5. Saturday 03/04 9-11 am **Midterm**; 12-2pm Lecture
6. Saturday 03/11 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
7. Saturday 03/18 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
8. Saturday 03/25 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
9. Saturday 04/01 **No class**
10. Saturday 04/08 9-11 am Lecture; 12-2pm Lab (Problem solving and coding)
11. Saturday 04/15 9-11 am Class Recap; 12-2pm **Final**

Grading and course policy

The grades of homework, exams, and quizzes will result in the final grade for the class according to:

- Homework 50%
- Quizzes 10%
- Midterm 20%
- Final 20%

Homework are due in class every Saturday at the *beginning of class*. No late homework will be accepted. You can collaborate as much as you want in completing your homework and discussion among students is encouraged. However, you must write your own solution and you cannot copy the solution from a classmate, a textbook, the Internet, etc. Regarding the coding exercise, you cannot copy and paste the code of a classmate, even partially. Your homework must reflect your own work and understanding, that is, you must be able to answer questions about the solution you turn in.

As part of your homework, you will be assigned one or more recorded lectures to watch before class. Before every class, there will be a short quiz to test your understanding of the recorded lectures.

There will be a two hours midterm and a two hours final. The exams will be closed books but you are allowed to write and bring with you a one page, double side summary with the formulas and examples you want.

The laboratory from 12 to 2 pm will also serve as office hours with the instructors and we will setup a website where students may discuss homework and the material presented in class.

Class notes and additional references

We will use the class notes prepared by Professor William S. Klug and Theodore A. Shugar. These class notes will be distributed to all students in pdf format. Additionally, the lectures recorded by Professor Klug will introduce the material presented in class every week.

The following textbooks provide in depth additional explanation of the material presented in class and advanced finite element topics for future reference. References are reported in order of difficulty.

1. The finite element method: linear static and dynamic finite element analysis. Hughes TJR, 2012, Dover.

2. The finite element method for solid and structural mechanics. Zienkiewicz OC and Taylor RL, 2005, Butterworth-heinemann.

3. Finite element procedures. Bathe KJ, 2006, Prentice Hall.

4. Finite elements: Theory, fast solvers, and applications in solid mechanics. Braess D, 2007, Cambridge University Press.

5. Theory and practice of finite elements, Ern A and Guermond JL, 2013, Springer Science.