Nobel

- pre-Bötzinger Complex ≈ 10^2 neurons
- rhythmic voltage signal sets the time of inspiration for mammals
- period of bursts ≈ 1 s, frequency of firing of individual neurons over 10^-2 s
- modulation is believed to be due to "adaptation", which is calcium-mediated
  
  Individual pacemaker hypothesis - there is a small number of pacemaker neurons, which control the process.

  Emergent pacemaker hypothesis - oscillation is a collective property. The oscillations in this case are expected to disappear if the number of neurons drops below some threshold, which happens in vitro when more than 80% of pBC neurons are destroyed.
In the paper, the authors consider the EP model. The results are that the general properties are defined by chemical connectivity characteristics, and specifically to the model as the following. All neurons are identical excitatory neurons. The equations are the following:

\[
\frac{dV_i}{dt} = \frac{1}{C_v} (V_{eq} - V_i) + \Delta V(C_i) \sum_{j \neq i} w_{ij} P(V_j)
\]

\[
\frac{dC_i}{dt} = \frac{1}{\tau_C} (C_{eq} - C_i) + \Delta C \sum_{j \neq i} w_{ij} P(V_j)
\]

\[V_{eq}\] : Resting potential and equilibrium calcium concentration with equilibration time \[\tau_v \approx 10 \text{ms} \quad \tau_C \approx 500 \text{ms}\]

\[\Delta V(C)\] drops rapidly when \(C\) exceeds threshold \(C^*\) (sigmoidal)

\[P(V)\] increases from \(\sim 5\) spikes per second to \(\sim 75\) spikes per second as \(V\) exceeds \(V^*\)
The simplest case: total connection, or clique. All $M_{ij} = 1$. Then we have

\[
\frac{dV}{db} = \frac{2}{\sigma_V} (V_{eq} - V) + N \Delta V(C) P(V)
\]

\[
\frac{dc}{db} = \frac{1}{\sigma_C} (C_{eq} - C) + N \Delta C P(C)
\]

The results are on Fig. 1.

More realistic model. With each neuron connected to $\frac{1}{m}$ of other neurons, assuming $M_{ij}$ is random, 0 with probability $\frac{5}{6}$ and 1 with probability $\frac{1}{6}$. The diagram changes a little bit.

Fig 1
low, for random matrices, the firing pattern for different neurons can be very different.

- Low activity - all below $V^*$
- Then for highly connected sub-population potentials start to rise, causing them to reach firing threshold, and then chain reaction along the whole network. Few threshold neurons are becoming emergent pacemakers. (all the neurons are created equal, but due to position in the network some become leaders)

- Deterministic chaos, period doubling in high activity phase
- SO-HA threshold has a staircase dependence on $\Delta V$

- Magic numbers of neurons as well their values are independent on system parameters as $D_C$. 
Selection of parameter neurons and values $N_k$ size are determined by $M_j$.

Paramaker - ones that reach maximal number of neurons, take no more than three connections. However, they do not play important role for $N_k$. They quickly turn "on" the system, but not "off".

$k$-core of a graph is a subgraph in which all nodes have at least $k$ inputs from other nodes in the subgraph.

$n = 40$ single 4-core cluster
$n = 41$ the same
$n = 42$ single 5-core cluster
$n = 43$ the same

$n_3 = 17 - 3$ core appear
$n_{45} = 26 - 4$ core appear
$n_5 = 34$

The locations of $N_3$ agree well but not perfectly with $k$-core dimension values.